

Chapter 4 Multiple-Choice Practice Test

Directions: This practice test features multiple-choice questions based on the content in Chapter 4: Integration.

4.1: Antidifferentiation

4.2: Definite Integrals

4.3: Fundamental Theorem of Calculus

4.4: Integration by Substitution

4.5: Numerical Integration

For each question, select the best answer provided and do your figuring in the margins. If you encounter difficulties with a question, then move on and return to it later. Follow these guidelines:

- Do not use a calculator of any kind. All of these problems are designed to contain simple numbers.
- Adhere to the time limit of 90 minutes.
- After you complete all the questions, compare your responses to the answer key on the last page. Note any topics that require revision.

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Integration

Number of Questions—50

NO CALCULATOR

1. How many antiderivatives does $f(x) = x^3 + 5x - 7$ have?

- (A) None
- (B) One
- (C) Two
- (D) Three
- (E) Infinitely many

2. $\frac{d}{dx} \int_{\pi/3}^x \cos t \, dt =$

(A) $\frac{1}{2}$

(B) $\sin x$

(C) $\cos x$

(D) $\sin x - \frac{\sqrt{3}}{2}$

(E) $\cos x - \frac{1}{2}$

3. For which of the following mathematical concepts is the result expected to include a constant of integration, C ?

- I. An indefinite integral
- II. A definite integral
- III. A derivative
- IV. A particular solution to a differential equation

- (A) I only
- (B) I and II only
- (C) I and IV only
- (D) I, II, and IV only
- (E) I, II, III, and IV

4. Ducks enter a pond at a rate given by $f(t)$, where t is the number of hours after midnight. What is the best interpretation of the equation $\int_0^5 f(t) dt = 60$?

(A) At 5 AM, there are 60 ducks in the pond.

(B) From 12 AM to 5 AM, 60 ducks enter the pond every hour.

(C) From 12 AM to 5 AM, the rate at which ducks enter the pond increases by 60 ducks per hour.

(D) From 12 AM to 5 AM, the pond always has 60 ducks.

(E) From 12 AM to 5 AM, 60 ducks enter the pond.

5. $\int_{-\pi/3}^{\pi/6} \cos x dx =$

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1-\sqrt{3}}{2}$ (D) $\frac{1+\sqrt{3}}{2}$ (E) $\frac{\sqrt{3}-1}{2}$

6. $\int \left(\sqrt[3]{x} - \cos x + \frac{1}{x} \right) dx =$

(A) $\frac{1}{3}x^{-2/3} + \sin x + \ln|x| + C$

(B) $\frac{1}{3}x^{-2/3} + \sin x - \ln|x| + C$

(C) $\frac{1}{3}x^{-2/3} - \sin x + \ln|x| + C$

(D) $\frac{3}{4}x^{4/3} + \sin x + \ln|x| + C$

(E) $\frac{3}{4}x^{4/3} - \sin x + \ln|x| + C$

7. $\int_1^2 (3x^2 + 6x + 4) dx =$

(A) 6

(B) 18

(C) 20

(D) 28

(E) 30

8. Which definite integral is equivalent to $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{1}{1 + \left(4 + \frac{2i}{n}\right)^3} \right) \frac{2}{n}$?

(A) $\int_0^2 \frac{1}{4+x^3} dx$

(B) $\int_0^4 \frac{1}{1+(4+x)^3} dx$

(C) $\int_4^6 \frac{1}{x^3} dx$

(D) $\int_4^6 \frac{1}{1+x^3} dx$

(E) $\int_4^6 \frac{1}{1+(4+x)^3} dx$

9. $\int_{-4}^3 5 dx =$

(A) -35

(B) -7

(C) -5

(D) 7

(E) 35

10. If $\frac{dy}{dt} = t^4 + 3t^2 - 8t + 9$, then

(A) $y = 12t^2 + 6 + C$

(B) $y = 4t^3 + 6t - 8 + C$

(C) $y = t^4 + 3t^2 - 8t + 9 + C$

(D) $y = \frac{1}{5}t^5 + t^3 - 4t^2 + 9t + C$

(E) $y = \frac{1}{30} + \frac{1}{4}t^4 - \frac{4}{3}t^3 + \frac{9}{2}t^2 + C$

11. $\int_4^{16} \frac{1}{x} dx =$

(A) $-\frac{3}{16}$

(B) $\frac{3}{16}$

(C) $\ln 4$

(D) $\ln 12$

(E) $\ln 64$

12. $\int_{-7}^7 \sqrt{49 - x^2} \, dx =$

- (A) $\frac{49\pi}{8}$ (B) $\frac{49\pi}{4}$ (C) $\frac{49\pi}{2}$ (D) 49π (E) 98π

13. $\int \frac{x-4}{x} \, dx =$

- (A) $-4\ln|x| + C$
(B) $x - \ln|x| + C$
(C) $x - 4\ln|x| + C$
(D) $x + 4\ln|x| + C$
(E) $\frac{x^2 - 8x}{x^2} + C$

14. $\int x^3 e^{x^4} dx =$

- (A) $\frac{1}{4}e^{x^4} + C$ (B) $e^{x^4} + C$ (C) $4e^{x^4} + C$ (D) $x^3 e^{x^4} + C$ (E) $\frac{e^{x^4}}{x^3} + C$

15. Let f be a twice-differentiable function. If $f(0) = 4$, $f(1) = 7$, and $f(4) = 8$, then $\int_0^1 f'(3x+1) dx =$

- (A) $\frac{1}{3}$ (B) 1 (C) 3 (D) 7 (E) 11

16. $\int x\sqrt[3]{x}dx =$

(A) $\frac{4}{3}\sqrt[3]{x} + C$

(B) $\frac{3}{4}\sqrt[3]{x^4} + C$

(C) $\sqrt[3]{x^4} + C$

(D) $\frac{3}{7}\sqrt[3]{x^7} + C$

(E) $\frac{3}{8}\sqrt[3]{x^{10}} + C$

17. If $g(1) = 8$ and $\int_4^1 g'(x) dx = 7$, where g' is continuous on $[1, 4]$, then what is the value of $g(4)$?

(A) -15

(B) -1

(C) 1

(D) 15

(E) It cannot be determined.

18. $\int \frac{1}{(x^2 + 1) \tan^{-1} x} dx =$

(A) $\ln(x^2 + 1) + C$

(B) $\ln|\tan^{-1} x| + C$

(C) $\frac{-1}{(\tan^{-1} x)^2} + C$

(D) $\tan^{-1} x + C$

(E) $\tan^{-1}(x^2 + 1) + C$

19. A stone is thrown downward with an initial speed of 20 meters per second from a position 10 meters above the ground. The stone's height above the ground as a function of time t , in seconds, is

(A) $-4.9t^2 + 20t + 10$

(B) $-4.9t^2 - 20t + 10$

(C) $-4.9t^2 + 20t - 10$

(D) $-4.9t^2 - 20t - 10$

(E) $4.9t^2 + 20t - 10$

20. $\int \frac{\tan^2 x}{\sec x \sin^2 x} dx =$

(A) $\ln |\cos x| + C$

(B) $\ln |\sec x| + C$

(C) $-\ln |\csc x| + C$

(D) $\ln |\sec x + \tan x| + C$

(E) $-\ln |\csc x + \cot x| + C$

21. What is the value of $\int_{-1}^1 \frac{x^4 \sin 3x}{\cos x} dx$?

(A) 0

(B) 2

(C) 4

(D) 8

(E) It cannot be determined analytically.

22. Let f be a continuous function such that $f(0) = 1$, $f(2) = 3$, and $f(4) = 6$. A left Riemann sum with two equal-width subintervals estimates $\int_0^4 f(x) \, dx$ to be

(A) 4 (B) 8 (C) 9 (D) 18 (E) 20

23. If $2 \leq f(x) \leq 8$ and f is integrable on $[1, 6]$, then

(A) $2 \leq \int_1^6 f(x) \, dx \leq 8$

(B) $10 \leq \int_1^6 f(x) \, dx \leq 40$

(C) $12 \leq \int_1^6 f(x) \, dx \leq 48$

(D) $14 \leq \int_1^6 f(x) \, dx \leq 56$

(E) $20 \leq \int_1^6 f(x) \, dx \leq 80$

24. $\int_0^1 (2x - 1)^4 dx =$

(A) $\frac{1}{40}$

(B) $\frac{1}{20}$

(C) $\frac{1}{10}$

(D) $\frac{1}{5}$

(E) $\frac{2}{5}$

25. $\frac{d}{dx} \int_2^{x^4-3x+2} \frac{1}{t} dt =$

(A) $\frac{1}{x^4 - 3x + 2}$

(B) $\frac{4x^3 - 3}{x^4 - 3x + 2}$

(C) $\ln|x^4 - 3x + 2|$

(D) $\ln|x^4 - 3x + 2| - \ln 2$

(E) $(4x^3 - 3) \ln|x^4 - 3x + 2|$

26. Selected values of the continuous function $f(x)$ are shown in the table below.

x	0	1	2	3	4
$f(x)$	1	5	7	2	-3

A midpoint approximation with two equal-width subintervals approximates $\int_0^4 f(x) \, dx$ to be

- (A) 7 (B) 8 (C) 12 (D) 14 (E) 16
27. Water begins entering a tank at a rate given by $f(t) = 3t^2 + t + 1$, where $f(t)$ is measured in gallons per minute and t is measured in minutes. If the tank initially has 50 gallons, then how many gallons of water are in the tank after 2 minutes?

- (A) 12 (B) 15 (C) 62 (D) 64 (E) 65

28. If $\int_2^4 g(x) \, dx = 7$ and $\int_2^6 g(x) \, dx = 4$, then $\int_4^6 g(x) \, dx =$

- (A) -11 (B) -3 (C) 3 (D) 11 (E) 28

29. Which function solves the initial value problem $\frac{dy}{dx} = 3x^2 - 6x + 2$, $y(0) = 7$?

- (A) $y = 6x - 6$
- (B) $y = 6x + 13$
- (C) $y = x^3 - 3x^2 + 2x$
- (D) $y = x^3 - 3x^2 + 5$
- (E) $y = x^3 - 3x^2 + 2x + 7$

30. By Simpson's Rule with $n = 4$, $\int_2^{10} \frac{1}{x} dx \approx$

- (A) $\frac{73}{45}$ (B) $\frac{73}{15}$ (C) $\frac{121}{90}$ (D) $\frac{121}{30}$ (E) $\frac{137}{180}$

31. Which set contains all the critical numbers of $f(x) = \int_1^x (t^3 - 8t^2 - 20t) dt$?

(A) $\left\{ -\frac{16 - \sqrt{496}}{6}, -\frac{16 + \sqrt{496}}{6} \right\}$

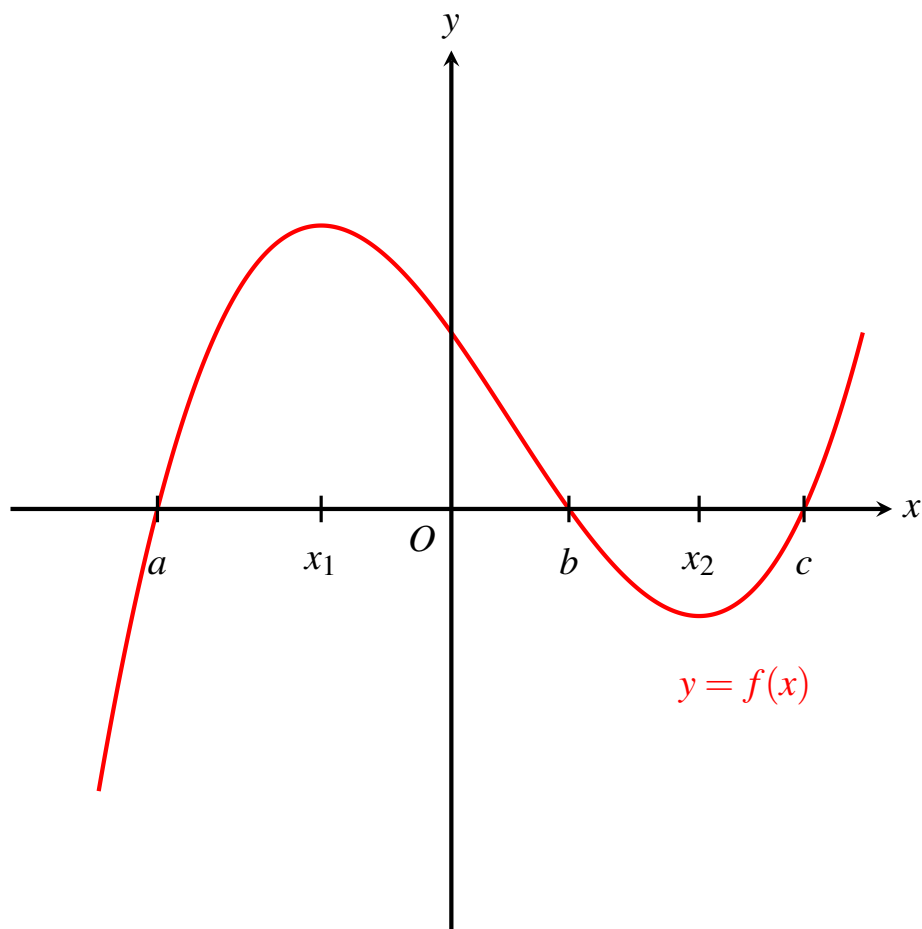
(B) $\left\{ \frac{16 - \sqrt{496}}{6}, \frac{16 + \sqrt{496}}{6} \right\}$

(C) $\{-2, 0, 10\}$

(D) $\{-10, 0, 2\}$

(E) $\emptyset$

Questions 32–34 refer to the following graph of f . Let $g(x) = \int_0^x f(t) \, dt$.



32. Which statement is true?

- (A) $\int_a^b f(x) \, dx < \int_a^c f(x) \, dx < \int_b^c f(x) \, dx$
- (B) $\int_a^b f(x) \, dx < \int_b^c f(x) \, dx < \int_a^c f(x) \, dx$
- (C) $\int_a^c f(x) \, dx < \int_b^c f(x) \, dx < \int_a^b f(x) \, dx$
- (D) $\int_b^c f(x) \, dx < \int_a^b f(x) \, dx < \int_a^c f(x) \, dx$
- (E) $\int_b^c f(x) \, dx < \int_a^c f(x) \, dx < \int_a^b f(x) \, dx$

33. g has a relative maximum at

- (A) $x = a$ (B) $x = x_1$ (C) $x = b$ (D) $x = x_2$ (E) $x = c$

34. On the interval $a \leq x \leq c$, the graph of g is concave up for x in

- (A) (a, b)
(B) (b, c)
(C) (x_1, x_2)
(D) (x_2, c)
(E) $(a, x_1) \cup (x_2, c)$

35. By Simpson's Rule with $n = 6$, $\int_0^\pi \sin x \, dx \approx$

(A) $\frac{\pi}{18} (2 + \sqrt{3})$

(B) $\frac{\pi}{9} (4 + \sqrt{3})$

(C) $\frac{\pi}{3} (4 + \sqrt{3})$

(D) $\frac{2\pi}{9} (1 + \sqrt{3})$

(E) $\frac{2\pi}{3} (1 + \sqrt{3})$

36. Which of the following statements are true about endpoint approximations of $I = \int_0^1 x^8 \sin x \, dx$?

- I. A left-endpoint approximation overestimates I .
- II. A right-endpoint approximation underestimates I .
- III. The Midpoint Rule is more effective than endpoint approximations in approximating I .

- (A) I only
- (B) II only
- (C) III only
- (D) I and II only
- (E) I, II, and III

37. The function $f(x) = \int_0^x (t^2 - 6t + 8) \, dt$ is decreasing on

- (A) $(-\infty, \infty)$
- (B) $(-\infty, 2] \cup [4, \infty)$
- (C) $(-\infty, 3]$
- (D) $[2, 4]$
- (E) $[3, \infty)$

38. $\int x\sqrt{3x+5} \, dx =$

(A) $\frac{1}{9}(3x+5)^{5/2} - \frac{5}{9}(3x+5)^{3/2} + C$

(B) $\frac{2}{45}(3x+5)^{5/2} - \frac{10}{27}(3x+5)^{3/2} + C$

(C) $\frac{2}{15}(3x+5)^{5/2} - \frac{10}{9}(3x+5)^{3/2} + C$

(D) $\frac{2}{5}(3x+5)^{5/2} - \frac{10}{3}(3x+5)^{3/2} + C$

(E) $\frac{6}{5}(3x+5)^{5/2} - 10(3x+5)^{3/2} + C$

39. Starting at $x = 2$, a particle travels along the x -axis with a velocity function given by $v(t) = 6t - 4$. When $t = 3$, the particle is located at

(A) $x = 13$ (B) $x = 14$ (C) $x = 15$ (D) $x = 16$ (E) $x = 17$

40. $\int \frac{1}{x^2 - 6x + 9} dx =$

(A) $-\frac{1}{x-3} + C$

(B) $\frac{1}{(x-3)^2} + C$

(C) $-\frac{1}{3(x-3)^3} + C$

(D) $-\ln|x-3| + C$

(E) $\ln[(x-3)^2] + C$

41. $\frac{d}{dx} \int_{x^2}^{\sin x} e^{4t^2} dt =$

(A) $e^{4\sin^2 x} - e^{4x^4}$

(B) $8xe^{4\sin^2 x} - 8xe^{4x^4}$

(C) $e^{4\sin^2 x} \sin x - x^2 e^{4x^4}$

(D) $e^{4\sin^2 x} \cos x - 2xe^{4x^4}$

(E) $8xe^{4\sin^2 x} \cos x - 16x^2 e^{4x^4}$

42. $\int x^3 \sqrt{x^2 + 4} \, dx =$

(A) $\frac{1}{6} (x^2 + 4)^3 - (x^2 + 4)^2 + C$

(B) $\frac{1}{5} (x^2 + 4)^{5/2} + \frac{4}{3} (x^2 + 4)^{3/2} + C$

(C) $\frac{1}{5} (x^2 + 4)^{5/2} - \frac{4}{3} (x^2 + 4)^{3/2} + C$

(D) $\frac{2}{5} (x^2 + 4)^{5/2} + \frac{8}{3} (x^2 + 4)^{3/2} + C$

(E) $\frac{2}{5} (x^2 + 4)^{5/2} - \frac{8}{3} (x^2 + 4)^{3/2} + C$

43. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(2 + \frac{3i}{n} \right) \frac{3}{n}$ is

(A) $\frac{7}{2}$

(B) 7

(C) $\frac{21}{2}$

(D) 21

(E) nonexistent

44. The Trapezoidal Rule with $n = 4$ approximates $\int_0^\pi \cos^2 x \, dx$ to be

- (A) $\frac{\pi}{8}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π (E) 2π

45. $\int \frac{1}{\sqrt{-x^2 - 12x}} \, dx =$

- (A) $-6 \sin^{-1} \left(\frac{x+6}{6} \right) + C$
(B) $-\frac{1}{6} \sin^{-1} \left(\frac{x+6}{6} \right) + C$
(C) $\frac{1}{6} \sin^{-1} \left(\frac{x+6}{6} \right) + C$
(D) $\sin^{-1} \left(\frac{x+6}{6} \right) + C$
(E) $6 \sin^{-1} \left(\frac{x+6}{6} \right) + C$

46. The Trapezoidal Rule with $n = 4$ is used to approximate $\int_1^4 \ln x \, dx$. An error bound for the estimate is

- (A) $\frac{9}{8}$ (B) $\frac{9}{16}$ (C) $\frac{9}{32}$ (D) $\frac{9}{64}$ (E) $\frac{9}{128}$

47. $\int \frac{x}{\sqrt{1-x^4}} \, dx =$

- (A) $\frac{1}{2} \sin^{-1}(x^2) + C$
(B) $\frac{1}{2} \tan^{-1}(x^2) + C$
(C) $\sin^{-1}(x^2) + C$
(D) $\tan^{-1}(x^2) + C$
(E) $2 \sin^{-1}(x^2) + C$

48. Simpson's Rule with $n = 8$ approximates $\int_0^\pi \cos 8x \, dx$ with an error bound given by
- (A) $\frac{\pi^3}{180}$ (B) $\frac{\pi^3}{90}$ (C) $\frac{\pi^5}{360}$ (D) $\frac{\pi^5}{180}$ (E) $\frac{\pi^5}{90}$
49. A left-endpoint approximation with $n = 4$ is used to approximate $\int_0^{\pi/4} \cos(32x^4) \, dx$. An error bound in the estimate is
- (A) $\frac{\pi^2}{32}$
- (B) $\frac{\pi^2}{64}$
- (C) $\frac{\pi^5}{32}$
- (D) $\frac{\pi^5}{64}$
- (E) $\frac{\pi^5}{128}$

50. $\lim_{x \rightarrow 0} \frac{1}{\sin x} \int_1^{\cos x} 2t \, dt$ is

- (A) -2 (B) -1 (C) 0 (D) 1 (E) nonexistent

This marks the end of the test. The following page contains the answers to all the questions.

- | | |
|-------|-------|
| 1. E | 33. C |
| 2. C | 34. E |
| 3. A | 35. B |
| 4. E | 36. C |
| 5. D | 37. D |
| 6. E | 38. B |
| 7. C | 39. E |
| 8. D | 40. A |
| 9. E | 41. D |
| 10. D | 42. C |
| 11. C | 43. C |
| 12. C | 44. C |
| 13. C | 45. D |
| 14. A | 46. D |
| 15. A | 47. A |
| 16. D | 48. D |
| 17. C | 49. D |
| 18. B | 50. C |
| 19. B | |
| 20. D | |
| 21. A | |
| 22. B | |
| 23. B | |
| 24. D | |
| 25. B | |
| 26. D | |
| 27. C | |
| 28. B | |
| 29. E | |
| 30. A | |
| 31. C | |
| 32. E | |