

Trigonometry

Trigonometric Ratios in a Right Triangle

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

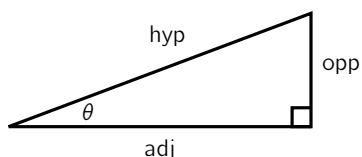
$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\cot \theta = \frac{\text{adj}}{\text{opp}}$$

$$\csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\sec \theta = \frac{\text{hyp}}{\text{adj}}$$



Quotient Identities

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

Periodic Properties

$$\sin x = \sin(x \pm 2\pi)$$

$$\cos x = \cos(x \pm 2\pi)$$

$$\csc x = \csc(x \pm 2\pi)$$

$$\sec x = \sec(x \pm 2\pi)$$

$$\tan x = \tan(x \pm \pi)$$

$$\cot x = \cot(x \pm \pi)$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\cot\left(\frac{\pi}{2} - x\right) = \tan x$$

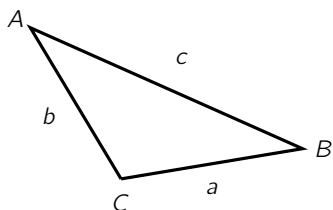
$$\csc\left(\frac{\pi}{2} - x\right) = \sec x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x$$

Law of Sines and Law of Cosines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Pythagorean Identity

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

Addition Identities

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Subtraction Identities

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Double-Angle Identities

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

Half-Angle Identities

$$\sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \cos x}{2}} \quad \cos\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \cos x}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

Power-Reduction Formulas

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

Product-to-Sum Identities

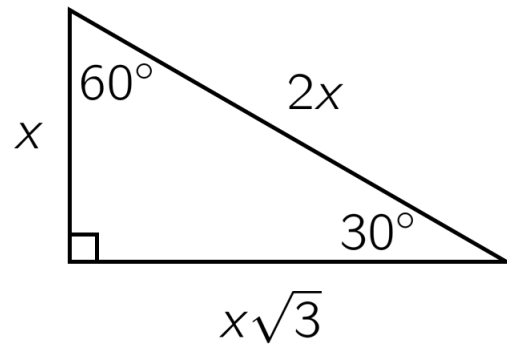
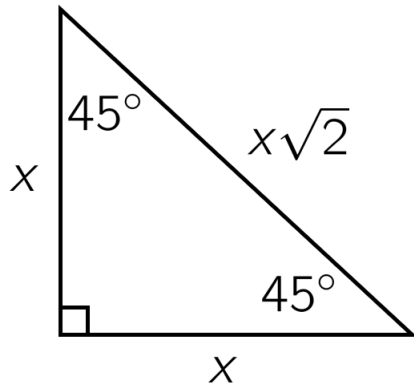
$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$$

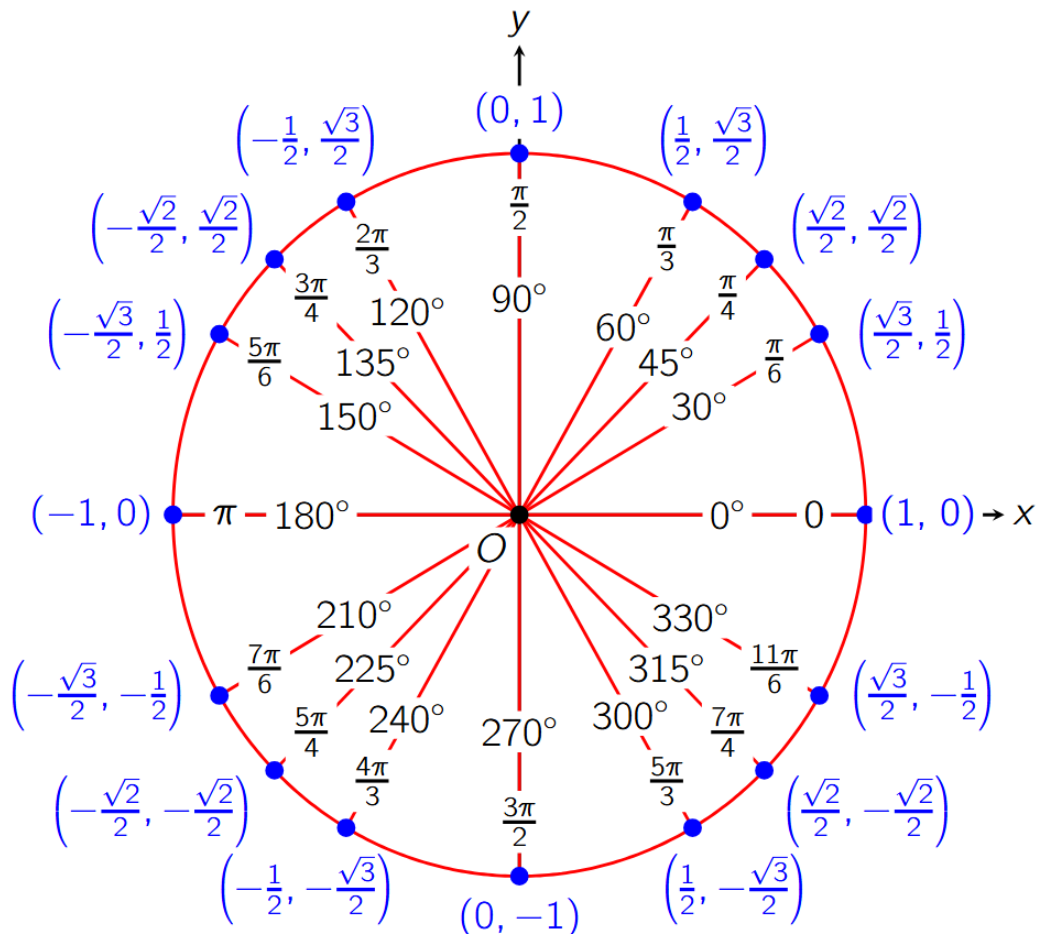
Trigonometry

Special Right Triangles



Unit Circle

$\pi \text{ rad} = 180^\circ$.



Trigonometry

Trigonometric Functions

Function	Domain*	Range	Period	Even or Odd?
$\sin x$	$(-\infty, \infty)$	$[-1, 1]$	2π	Odd
$\cos x$	$(-\infty, \infty)$	$[-1, 1]$	2π	Even
$\csc x$	$\{x \mid x \neq n\pi\}$	$(-\infty, -1] \cup [1, \infty)$	2π	Odd
$\sec x$	$\{x \mid x \neq \frac{\pi}{2} + n\pi\}$	$(-\infty, -1] \cup [1, \infty)$	2π	Even
$\tan x$	$\{x \mid x \neq \frac{\pi}{2} + n\pi\}$	$(-\infty, \infty)$	π	Odd
$\cot x$	$\{x \mid x \neq n\pi\}$	$(-\infty, \infty)$	π	Odd

* n is any integer.

Inverse Trigonometric Functions

Function	Domain	Range
$\sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$